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PASSAGE OF AN ELECTROMAGNETIC PULSE THROUGH A LAYER OF HOMOGENEOUS PLASMA (EXACT SOLUTION)

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The authors found an exact analytical solution of the problems on reflected and transmitted waves in interaction of a short electromagnetic pulse with a plasma layer of finite extension. The problem is solved assuming the linearity of material equations. Analytical expressions of Green functions are obtained for the corresponding problems that allow one to write solutions for the transmitted and reflected waves with any shape of an incident wave. The exact solutions depend substantially on the plasma density and layer extension, which makes it possible to use them in processing experimental results for extraction of the above-mentioned parameters.

Introduction. Recent advances in laser technology have enabled the production of high-power ultrashort pulses, by means of which it is possible to virtually instantly ionize thin layers of a substance. These can be both foils of a solid substance and gas jets injected into vacuum, so that the range of densities of plasmas generated turns out to be rather extensive. The properties of such plasmas are investigated by means of sounding pulses of low power, whose propagation can be considered in a linear regime. In diagnosing, of great interest is the passage of short pulses (the plasma formed must have no time to fly apart) through the thin layers; moreover, the carrier frequency must be close to a plasma frequency. The latter implies that dispersion of the waves in the plasma should be strictly taken into account.

On the other hand, the comparatively small transverse dimension of the plasmas formed requires the correct consideration of boundary conditions on both edges of the plasma spacing. The problems of interaction of ultrashort pulses with thin layers of a substance were investigated in [1, 2]. The present work is devoted to a rigorous solution of the problem concerning the passage of a short laser pulse of arbitrary shape through thin plasma layers. A similar problem was solved in [3] but for a semiinfinite plasma, i.e., only one boundary condition was taken into account. Taking into account the second boundary not only complicates the solution, but also requires the elucidation of what kind of contribution the effects of rereflections make to the transmitted and reflected field and what the influence of the plasma-layer extension is.

We express the solution in terms of Green functions for the transmitted and reflected waves without restricting ourselves to a specific shape of the envelope of an incident pulse. In our approach, the expansion in terms of the so-called nonseparable solutions [4] of the wave equation naturally arises.

Statement of the Problem. An electromagnetic pulse is incident from infinity onto a layer of homogeneous isotropic plasma of finite extension. The pulse is normally incident onto the leading edge of the plasma (along the X axis); the electric field of the wave is linearly polarized. We consider a one-dimensional problem, i.e., there is no dependence of the dielectric permeability on the transverse coordinates Y and Z. The plasma is described in a linear approximation and has the dielectric permeability

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}. \tag{1}$$

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In vacuum, the fields $E_r(t, x)$ and $E_{out}(t, x)$ satisfy an ordinary homogeneous wave equation, while the field $E_t(t, x)$ in the plasma layer obeys the equation

$$\left(\partial^2_{xx} - \frac{1}{c^2}\partial^2_{tt} - \frac{\omega_p^2}{c^2}\right)E_t(t, x) = 0\,, \tag{2}$$

which follows from Maxwell equations for a medium with dielectric permeability (1). Here we use the following notation: ∂^2_{xx} and ∂^2_{tt} are the partial derivatives of the second order with respect to the spatial coordinate x and time t, respectively; ω_p is the plasma frequency; c is the velocity of light in vacuum.

The boundary conditions on the electric field follow from the continuity conditions for the electric and magnetic fields at both boundaries of the layer ($x = 0$ and $x = l$) (it is assumed that surface charges and currents are absent):

$$E_i(t, 0) + E_r(t, 0) = E_t(t, 0)\,,\quad \partial_x E_i(t, 0) + \partial_x E_r(t, 0) = \partial_x E_t(t, 0)\,,$$
$$E_t(t, l) = E_{out}(t, l)\,,\quad \partial_x E_t(t, l) = \partial_x E_{out}(t, l)\,. \tag{3}$$

At the initial instant of time, the pulse $E_i(t - x)$ is incident onto the leading edge of the plasma; the latter is in an unperturbed state, therefore, the initial conditions are as follows:

$$E_i(0, x) = E_i(0 - x) = E_i(-x)\,,\quad E_t(0, x) = 0\,,\quad \partial_t E_t(0, x) = 0\,. \tag{4}$$

Solution. Let us pass to the dimensionless variables: $\omega_p t \to t$ and $\omega_p \frac{x}{c} \to x$. Then wave equation (2) takes the form

$$(\partial^2_{xx} - \partial^2_{tt} - 1)\,E_t(t, x) = 0\,. \tag{5}$$

To solve the last differential equation, we use the Laplace operator method. Using initial conditions (4) and taking into account the transformation properties that refer to the original time derivative, we obtain an equation equivalent to Eq. (5):

$$[d^2_{xx} - (s^2 + 1)]\,\overline{E}_t(s, x) = 0\,. \tag{6}$$

We represent the solution of Eq. (6) in the form

$$\overline{E}_t(s, x) = \overline{E}_t^{\,1}(s, 0)\exp(-\sqrt{s^2+1}\,x) + \overline{E}_t^{\,2}(s, 0)\exp(\sqrt{s^2+1}\,x)\,. \tag{7}$$

On the Laplace transform, boundary conditions (3) become as follows:

$$\overline{E}_i(s, 0) + \overline{E}_r(s, 0) = \overline{E}_t(s, 0)\,,\quad \overline{E}_t(s, l) = \overline{E}_{out}(s, l)\,,$$
$$\partial_x \overline{E}_i(s, 0) + \partial_x \overline{E}_r(s, 0) = \partial_x \overline{E}_t(s, 0)\,,\quad \partial_x \overline{E}_t(s, l) = \partial_x \overline{E}_{out}(s, l)\,. \tag{8}$$

Now we find the space-time dependences of the fields that propagate in vacuum. Since the incident pulse $E_i(t, x)$ moves from left to right (in the direction to the plasma), whereas the reflected pulse $E_r(t, x)$ moves from right to left (from the plasma), and finally the transmitted pulse $E_{out}(t, x)$ moves to the right, we have the dependences

$$E_i(t, x) = E_i(t - x)\,,\quad E_r(t, x) = E_r(t + x)\,,\quad E_{out}(t, x) = E_{out}(t - x)\,, \tag{9}$$

which correspond to the following wave equations:

$$(\partial_t + \partial_x) E_i (t, x) = 0 \,, \; (\partial_t - \partial_x) E_r (t, x) = 0 \,, \; (\partial_t + \partial_x) E_{out} (t, x) = 0 \,. \tag{10}$$

We take the Laplace transforms of the last equations, then determine the limiting transitions $x \to 0$ in the first two equations and $x \to l$ in the third of them, and find expressions for the field derivatives at the boundaries:

$$\begin{gathered} d_x \bar{E}_i (s, 0) = - s\bar{E}_i (s, 0) \,, \\ d_x \bar{E}_r (s, 0) = s\bar{E}_r (s, 0) \,, \; d_x \bar{E}_{out} (s, l) = - s\bar{E}_{out} (s, l) \,. \end{gathered} \tag{11}$$

Substitution of Eq. (7) into Eq. (8) with account for conditions (11) gives the solution for the transmitted wave

$$\begin{gathered} \bar{E}_{out} (s, l) = \frac{4\alpha s}{(s + \alpha)^2 \exp (\alpha l) - (s - \alpha)^2 \exp (- \alpha l)} \bar{E}_i (s, 0) \,, \\ \alpha = \sqrt{s^2 + 1} \,. \end{gathered} \tag{12}$$

From the third equation of (11), it follows that

$$\bar{E}_{out} (s, x) = \bar{E}_{out} (s, l) \exp (- s (x - l)) \,, \tag{13}$$

and then

$$\bar{E}_{out} (s, x) = \frac{4\alpha s \exp (- s (x - l))}{(s + \alpha)^2 \exp (\alpha l) - (s - \alpha)^2 \exp (- \alpha l)} \bar{E}_i (s, 0) \,. \tag{14}$$

To find $E_{out}(t, x)$, we take the inverse Laplace transform

$$E_{out} (t, x) = \frac{1}{2\pi i} \int_{-i\infty}^{+i\infty} \bar{E}_{out} (s, x) \exp (st) \, ds \,. \tag{15}$$

Using the determination

$$\bar{E}_i (s, 0) = \int_0^{\infty} E_i (\tau, 0) \exp (- s\tau) \, d\tau \,, \tag{16}$$

we obtain

$$E_{out} (t, x) = \frac{1}{2\pi i} \int_{-i\infty}^{+i\infty} \frac{4\alpha s \exp [s (t - (x - l))]}{(s + \alpha)^2 \exp (\alpha l) - (s - \alpha)^2 \exp (- \alpha l)} \int_0^{\infty} E_i (\tau, 0) \exp (- s\tau) \, d\tau \, ds \,. \tag{17}$$

It is convenient to represent the solution of the initial problem in the form of a convolution of the pulse at inlet with the Green function $G_{out}(t - \tau, x)$; to do this, we perform the rearrangement of the integration variables in the previous integral and introduce by definition

$$\begin{gathered} G_{out} (t - \tau, x) = \frac{1}{2\pi i} \int_{-i\infty}^{+i\infty} \frac{4\alpha s \exp (su)}{(s + \alpha)^2 \exp (\alpha l) - (s - \alpha)^2 \exp (- \alpha l)} \, ds \,, \\ u = t - \tau - (x - l) \,. \end{gathered} \tag{18}$$

Then finally the field of the transmitted pulse can be represented in the form of the convolution

$$E_{\text{out}}(t,x)=\int_0^{\infty} G_{\text{out}}(t-\tau,x)\,E_{\text{i}}(\tau,0)\,d\tau\,. \tag{19}$$

To find the field of the reflected pulse, we express $\bar{E}_{\text{r}}(s,0)$ from boundary conditions (8) and take into account Eq. (11):

$$\bar{E}_{\text{r}}(s,0)=\frac{\exp(-\alpha l)-\exp(\alpha l)}{(s+\alpha)^2\exp(\alpha l)-(s-\alpha)^2\exp(-\alpha l)}\,\bar{E}_{\text{i}}(s,0)\,. \tag{20}$$

From the second equation of (11) we find that

$$\bar{E}_{\text{r}}(s,x)=\bar{E}_{\text{r}}(s,0)\exp(sx)\,. \tag{21}$$

Introducing the Green function of the reflected wave

$$G_{\text{r}}(t-\tau,x)=\frac{1}{2\pi i}\int_{-i\infty}^{+i\infty}\frac{(\exp(-\alpha l)-\exp(\alpha l))\exp(sv)}{(s+\alpha)^2\exp(\alpha l)-(s-\alpha)^2\exp(-\alpha l)}\,ds\,,$$
$$v=(t-\tau+x)\,, \tag{22}$$

we can also represent the field $E_{\text{r}}(t,x)$ in the form of the convolution

$$E_{\text{r}}(t,x)=\int_{-i\infty}^{+i\infty} G_{\text{r}}(t-\tau,x)\,E_{\text{i}}(\tau,0)\,d\tau\,. \tag{23}$$

The problem is reduced to taking the integrals in formulas (18) and (22).

We will illustrate in detail the integration process for the Green function of the transmitted field $G_{\text{out}}(t-\tau,x)$; for $G_{\text{r}}(t-\tau,x)$ everything is similar. In integrals (18) and (22), the contour of integration represents a straight line parallel to an imaginary axis. We introduce a new variable:

$$w=s+\sqrt{s^2+1}\,. \tag{24}$$

The old variables s and α will be expressed in terms of the variable w:

$$s=\frac{1}{2}\left(w-\frac{1}{w}\right),\quad \sqrt{s^2+1}=\frac{1}{2}\left(w+\frac{1}{w}\right),\quad ds=\frac{1}{2}\left(1+\frac{1}{w^2}\right)dw\,. \tag{25}$$

Now the Green function $G_{\text{out}}(t-\tau,x)$ is represented by the new integral:

$$G_{\text{out}}(t-\tau,x)=\frac{1}{2\pi i}\int_{\tilde{C}}\frac{1}{2}\,\frac{\left(w-\frac{1}{w}\right)\left(w+\frac{1}{w}\right)^2\exp\left[\frac{1}{2}\left(w-\frac{1}{w}\right)u\right]}{w^3\exp\left[\frac{1}{2}\left(w+\frac{1}{w}\right)l\right]-w^{-1}\exp\left[-\frac{1}{2}\left(w+\frac{1}{w}\right)l\right]}\,dw\,, \tag{26}$$

where $\tilde{C}$ is the new integration contour. We rewrite the first transformation in Eq. (25) in the equivalent form

$$is=\frac{1}{2}\left(iw+\frac{1}{iw}\right); \tag{27}$$

the right-hand side of Eq. (27) is a Zhukovskii function. From the form of the initial contour and the properties of the Zhukovskii function we obtain that the contour $\tilde{C}$ is an aggregate of two beams on the imaginary axis $(-i\infty, -i)$, $(i, i\infty)$ and of a semicircle of unity radius in the right half-plane.

We consider the denominator of the integrand of Green function (26)

$$w^3 \exp\left(\frac{1}{2}\left(w+\frac{1}{w}\right)l\right) - w^{-1} \exp\left(-\frac{1}{2}\left(w+\frac{1}{w}\right)l\right). \tag{28}$$

and transform it to the form

$$w^3 \exp\left(\frac{1}{2}\left(w+\frac{1}{w}\right)l\right)\left[1 - w^{-4} \exp\left(-2\frac{1}{2}\left(w+\frac{1}{w}\right)l\right)\right]. \tag{29}$$

Since on the new integration contour $|w| \geq 1$ and

$$\left|\left(w+\frac{1}{w}\right)\right| \geq 1\,, \tag{30}$$

we expand the expression in square brackets in Eq. (29) into a power series:

$$\frac{1}{1 - w^{-4} \exp\left(-2\frac{1}{2}\left(w+\frac{1}{w}\right)l\right)} = \sum_{k=0}^{\infty} \left[w^{-4} \exp\left(-2\frac{1}{2}\left(w+\frac{1}{w}\right)l\right)\right]^k ; \tag{31}$$

then

$$G_{\text{out}}(t-\tau, x) = \sum_{k=0}^{\infty} \frac{1}{2\pi i} \int_{\tilde{C}} \frac{1}{2} \frac{1}{w^{4k+3}} \exp\left(w - \frac{1}{w}\right)\left(w+\frac{1}{w}\right)^2 \times$$
$$\times \exp\left[\frac{1}{2}\left(w-\frac{1}{w}\right)u - \frac{1}{2}(2k+1)\,l\left(w+\frac{1}{w}\right)\right]dw\,. \tag{32}$$

The integral over the infinitely distant circle diverges for a term of sum with $k = 0$; therefore, in order to close the contour $\tilde{C}$ at infinity, we represent $G_{\text{out}}(t-\tau, x)$ in the form of the derivative of a certain function $\tilde{G}_{\text{out}}(t-\tau, x)$ with respect to the parameter u (see Eq. (18)):

$$G_{\text{out}}(t-\tau, x) = \frac{\partial}{\partial u} \tilde{G}_{\text{out}}(t-\tau, x)\,, \tag{33}$$

where

$$\tilde{G}_{\text{out}}(t-\tau, x) = \sum_{k=0}^{\infty} \frac{1}{2\pi i} \int_{\tilde{C}} \frac{1}{2} \frac{1}{w^{4k+3}} \left(w+\frac{1}{w}\right)^2 \exp\left[\frac{1}{2}\left(w-\frac{1}{w}\right)u\right] \times$$
$$\times \exp\left[-\frac{1}{2}(2k+1)\,l\left(w+\frac{1}{w}\right)\right]dw\,. \tag{34}$$

Now the integrand in Eq. (34) vanishes when $|w| \to \infty$ and the contour $\tilde{C}$ can be closed, but in the left half-plane on the condition that $u - (2k+1)l > 0$; otherwise the integral vanishes, since the contour will not contain special features of the integrand function.

We try to eliminate such a complex expression of the argument in the exponential. For this, we introduce a new variable ξ and the parameter y_k:

$$y_k = (2k+1)\,l\,, \quad w = \xi \left(\frac{u+y_k}{u-y_k}\right)^{\frac{1}{2}}, \quad dw = d\xi \left(\frac{u+y_k}{u-y_k}\right)^{\frac{1}{2}}. \tag{35}$$

The argument of the exponential in Eq. (34) will be rewritten in terms of the new variable ξ and parameter y_k:

$$\frac{1}{2}\left(w-\frac{1}{w}\right)u - y_k\left(w+\frac{1}{w}\right)\frac{1}{2} = \frac{1}{2}\,\mu_k\left(\xi-\frac{1}{\xi}\right), \quad \mu_k = \sqrt{u^2-y_k^2}\,; \tag{36}$$

then

$$\widetilde{G}_{\text{out}}\,(t-\tau, x) = \frac{1}{2\pi i}\int\limits_{\widetilde{C}} \frac{1}{2}\left(\frac{u-y_k}{u+y_k}\right)^{2k}\left(\frac{1}{\xi}+\frac{2}{\xi^3}\left(\frac{u-y_k}{u+y_k}\right)+\frac{1}{\xi^5}\left(\frac{u-y_k}{u+y_k}\right)^2\right)d\xi \times$$

$$\times \exp\left[\frac{1}{2}\,\mu_k\left(\xi-\frac{1}{\xi}\right)\right]\sum_{k=0}^{\infty}\frac{1}{\xi^{4k}}\,. \tag{37}$$

Now we make use of the fact that $\exp\left[\frac{1}{2}\,\mu_k(\xi-\frac{1}{\xi})\right]$ is the Bessel generating function

$$\exp\left[\frac{1}{2}\,\mu_k\left(\xi-\frac{1}{\xi}\right)\right] = \sum_{m=-\infty}^{\infty}\xi^m J_m\,(\mu_k)\,; \tag{38}$$

then $\widetilde{G}_{\text{out}}(t-\tau, x)$ takes the form

$$\widetilde{G}_{\text{out}}\,(t-\tau, x) = \frac{1}{2\pi i}\int\limits_{\widetilde{C}} \left(\frac{u-y_k}{u+y_k}\right)^{2k}\left(\frac{1}{\xi}+\frac{2}{\xi^3}\,\frac{u-y_k}{u+y_k}+\frac{1}{\xi^5}\left(\frac{u-y_k}{u+y_k}\right)^2\right)d\xi \times$$

$$\times \sum_{m=-\infty}^{\infty}\xi^m J_m\,(\mu_k)\sum_{k=0}^{\infty}\frac{1}{\xi^{4k}}\,. \tag{39}$$

Next, we integrate Eq. (39), taking into account that

$$\frac{1}{2\pi i}\oint\limits_{\gamma}\frac{1}{\xi^n}\,d\xi = \begin{cases} 1\,, & n=1\,,\\ 0\,, & n\neq 1\,,\end{cases} \tag{40}$$

where γ is the arbitrary closed contour that covers the point $\xi = 0$. As a result, we obtain the Green function of the reflected wave

$$\widetilde{G}_{\text{out}}\,(t-\tau, x) = \sum_{k=0}^{\infty}\left(\frac{u-y_k}{u+y_k}\right)^{2k}\left(J_{4k}\,(\mu_k) + 2\,\frac{u-y_k}{u+y_k}\,J_{4k+2}\,(\mu_k) + \right.$$

$$+\frac{(u-y_k)^2}{(u+y_k)^2}J_{4k+4}(\mu_k)\Bigg)\theta(u-y_k)\,. \tag{41}$$

Upon differentiation, we write the final expression

$$G_{\text{out}}(t-\tau,x)=\delta(u-y_0)+4\sum_{k=0}^{\infty}\left(\frac{u-y_k}{u+y_k}\right)^{2k-1}\left(\frac{k}{u+y_k}J_{4k}(\mu_k)-(2k+1)\times\right.$$

$$\times y_k\frac{(u-y_k)^2}{(u+y_k)^4}J_{4k+2}(\mu_k)-(k+1)\frac{(u-y_k)^3}{(u+y_k)^4}J_{4k+4}(\mu_k)-$$

$$\left.-\frac{uy_k^2}{\mu_k}\frac{u-y_k}{(u+y_k)^3}J_{4k+1}(\mu_k)\right)\theta(u-y_k)\,, \tag{42}$$

$$u=t-\tau+(x-l)\,,\ \ y_k=(2k+1)\,l\,,\ \ \mu_k=\sqrt{u^2-y_k^2}\,.$$

To find $G_r(t-\tau,x)$, we perform the same operations but taking into account that in this case the integration contour can be closed for the integrand function itself.

The Green function of the reflected field is as follows:

$$G_r(t-\tau,x)=-\frac{2}{v}J_2(v)\,\theta(v)+\frac{1}{2}\sum_{k=1}^{\infty}\left(\frac{v-b_k}{v+b_k}\right)^{2k-\frac{1}{2}}\left(-\frac{(v-b_k)}{(v+b_k)}J_{4k+1}(\nu_k)-\right.$$

$$\left.-\frac{(v-b_k)^2}{(v+b_k)^2}J_{4k+3}(\nu_k)+\frac{v+b_k}{v-b_k}J_{4k-3}(\nu_k)+J_{4k-1}(\nu_k)\right)\theta(v-b_k)\,, \tag{43}$$

$$v=t-\tau+x\,,\ \ b_k=2kl\,,\ \ \nu_k=\sqrt{v^2-y_k^2}\,.$$

Now we analyze expressions (42) and (43) and explain the physical meaning of separate terms. The solutions consist of an infinite series of terms that describe numerous rereflections from both boundaries. However, at any finite instant of time the number of terms is finite because of the presence of the θ functions.

As expected, expression (42) for the linear medium represents a sum of the initial pulse (the δ-function) and of the disturbing term (the sum over k). The meaning of the terms of the disturbing term is quite obvious, namely, each kth term is the contribution to the transmitted pulse that appears as a result of the kth reflection from the leading and rear boundary of the layer, during which the δ-function and the term with k = 0 describe a single passage of the initial pulse through the plasma. The terms in Eq. (43) have a similar meaning.

Numerical Illustration of the Results. We apply the theory described for a particular case when the incident wave is a Gaussian pulse:

$$E_i(t,x)=\exp\left(-\omega_d^2(t-x)^2\right)\cos\left(\omega_c(t-x)\right)\,. \tag{44}$$

The parameters ω_d and ω_c are the measures of the duration and carrier frequency of the pulse, respectively.

We consider the case of short pulses (of 5–6 oscillations). The character of the reflected and transmitted waves depends substantially on the relationship between the carrier and plasma frequencies as well as on

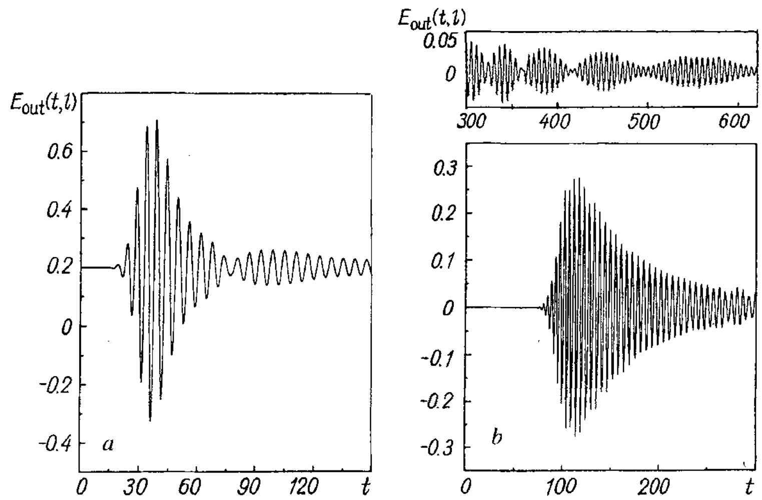

Fig. 1. Time dependence of the transmitted pulse $E_{out}(t, l)$ on the rear edge of the plasma at incidence of the initial pulse $E_i(t, x)$ with the parameters (the frequency is measured in units of ω_p, the lengths are measured in wavelengths of the incident pulse, the amplitudes of the fields are given in arbitrary units): a) carrier frequency $\omega_c = 1.1$ and duration of the order of 5–6 oscillations of the field, the plasma thickness $l = 2$; b) parameters $\omega_c = 1.1$ and $l = 10$, the continuation of the graph for large times is given above.

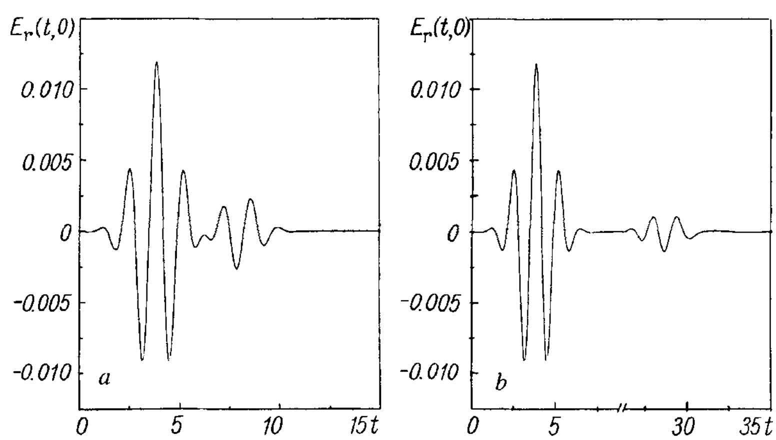

Fig. 2. Time dependence of the reflected pulse $E_r(t, 0)$ on the leading edge of the plasma at incidence of the initial pulse $E_i(t, x)$ with the parameters (for the units of measurement, see Fig. 1): a) parameters $\omega_c = 5$ and $l = 2$; one can see the pulse that is reflected from the leading edge and the second pulse reflected from the rear edge; b) parameters $\omega_c = 5$ and $l = 10$.

the plasma-spacing length. At the frequency $\omega_c \to \omega_p$ the plasma dispersion is considerable, and therefore the distortions of the pulse are significant.

To evaluate and analyze the solutions qualitatively, we consider some cases:

(1) $\omega_c \sim \omega_p$, $l \sim 1$, and $l \sim 10$;

(2) $\omega_c >> \omega_p$, $l \sim 1$, and $l \sim 10$.

Let us pass to analysis of the results obtained. Case (1) is represented in Fig. 1 by several examples. As a consequence of strong dispersion, a characteristic feature is the distortion of the transmitted pulse and its time delay relative to the initial pulse retransmitted at the point $x = l$. The time delay can be explained by the fact that the group velocity of the wave packet in the plasma is smaller than the velocity of light in vacuum. Further, we note that in contrast to the case of a semiinfinite layer, the effects of rereflection from both boundaries are manifested, which make a considerable contribution to the transmitted and reflected pulses.

Case (2) (Fig. 2a) is characterized by a higher carrier frequency and, as a result, a smaller dispersion. The influence of the plasma-layer length is manifested more weakly for the reflected pulse at a layer thickness smaller than the extension of the initial pulse and more strongly for the opposite case (Fig. 2b). For the pulses passed through the plasma the rereflection effects do not contribute significantly.

Conclusions. The interaction of the plane wave of a short electromagnetic pulse with the plasma layer of finite extension has been considered. It was shown that the solutions of the posed problem for the transmitted and reflected waves differ significantly from those in the case of interaction with a semiinfinite layer. This difference is expressed in the appearance of rereflections at both plasma–vacuum interfaces and in a more considerable dependence of the fields on the plasma density and plasma-spacing length.

The analytical dependence of the shape of the pulse of the transmitted and reflected waves on the indicated parameters of the plasma layer makes it possible to use the solutions obtained for evaluating the latter by experimental information using correlation or spectral data.

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NOTATION

ω, cyclic frequency; ω_p, plasma frequency; c, velocity of light in vacuum; l, length of plasma spacing; variables x and t, coordinate along the X axis and time, respectively; E_i, E_r, and E_{out}, electric fields of incident, reflected, and transmitted pulses; E_t, electric field of the wave in a plasma layer; $\bar{E}_i$, $\bar{E}_r$, $\bar{E}_t$, and $\bar{E}_{out}$, images of the corresponding fields in the Laplace space; G_r, G_{out}, Green functions of the reflected and transmitted waves, respectively; J_m, Bessel function of the mth order; δ, Dirac delta-function; θ, Heaviside function; ω_d and ω_c, measure of duration and the carrier frequency of the pulse; s, τ, w, α, ξ, μ_k, y_k, v_k, b_k, u, and v, intermediate variables. Subscripts: i, incident; r, reflected; t, incoming; out, outgoing; c, carrier; d, duration; k and m, indices of the series numbering.

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